SAE 1

1. A beam has a bending moment (M) of 3 kNm applied to a section with a second moment of area (I) of 5 x 10⁻³ m⁴. The modulus of elasticity for the material (E) is 200 x 10⁹ N/m². Calculate the radius of curvature. (Answer 333.3 km).

SOLUTION

\[ M = 3000 \text{ Nm} \quad I = 0.005 \text{ m}^4 \quad E = 200 \text{ GPa} \]

\[ R = \frac{EI}{M} = \frac{200 \times 10^9 \times 0.005}{3000} = 333.3 \times 10^3 \text{ m} \]

2. The beam is Q1 has a distance from the neutral axis to the edge in tension of 60 mm. Calculate the stress on the edge. (Answer 36 kPa).

SOLUTION

\[ \sigma = \frac{My}{I} = \frac{3000 \times 0.06}{0.005} = 36000 \text{ N/m}^2 \text{ or } 36 \text{ kPa} \]

3. A beam under test has a measured radius of curvature of 300 m. The bending moment applied to it is 8 Nm. The second moment of area is 8000 mm⁴. Calculate the modulus of elasticity for the material. (Answer 300 GPa).

SOLUTION

\[ E = \frac{MR}{I} = \frac{8 \times 300}{8000 \times 10^{-12}} = 300 \times 10^9 \text{ N/m}^2 \text{ or } 300 \text{ GPa} \]

SAE 2

1. A symmetrical I section beam is 60 mm deep with a second moment of area of 663 x 10⁻⁹ m⁴ and a cross sectional area of 1600 mm². It is subject to a bending moment of 1.2 kNm and an axial force of 25 kN (tension). Find the position of the neutral axis. (Answer the stresses are 69.92 MPa and -38.67 MPa and the neutral axis is 38.6 mm from the tensile edge)

SOLUTION

\[ y = 60 \text{ mm} \]

Bending Stress \( \sigma_b = \frac{My}{I} = \pm1200 \times 0.03/663 \times 10^{-9} = \pm54.3 \text{ MPa} \)

Direct Stress = \( F/A = 25000/1600 \times 10^{-3} = 15.62 \text{ MPa} \)

Maximum tensile stress = 54.3 + 15.62 = 69.92 MPa

Maximum compressive stress = -54.3 + 15.62 = -38.67 MPa

\[ \frac{A}{69.92} = \frac{B}{38.67} \quad A = 1.808 \times B \quad A + B = 60 \quad B = 60 - A \]

\[ A = 1.808(60-A) = 108.49 - 1.808 \times A \quad 2.808 \times A = 108.49 \quad A = 38.6 \text{ mm} \]
SAE 3

1. A rectangular section timber beam is 60 mm wide and 100 mm deep. It is clad with steel plate 12 mm thick on the top and bottom. Calculate the maximum stress in the steel and the timber when a moment of 5 kNm is applied.
   E for timber is 11 GPa and for steel 205 GPa.

SOLUTION

Equivalent width of steel for timber \( t = B \left( \frac{E_t}{E_s} \right) = 60 \times \left( \frac{11}{205} \right) = 3.219 \text{ mm} \)

\[
I_{gg} = BD^3/12 - bd^3/12 = 60 \times 124^3/12 - 56.8 \times 75^3/12 = 9.533 \times 10^6 - 4.731 \times 10^6 = 4.8 \times 10^6 \text{ mm}^4
\]

\[
I_{gg} = 4.8 \times 10^{-6} \text{ m}^4
\]

For the timber \( y = 50 \text{ mm} \) \( \sigma = My/I = 5000 \times 0.05 / 4.8 \times 10^{-6} = 52 \text{ MPa} \) but needs converting.

\( \sigma \text{ (timber)} = 24.88 \times \left( \frac{E_t}{E_s} \right) = 28 \text{ MPa} \)

For the steel \( y = 62 \text{ mm} \) \( \sigma = My/I = 5000 \times 0.062 / 4.8 \times 10^{-6} = 64.5 \text{ MPa} \)
2. A symmetrical steel I section beam has a second moment of area $I_{gg} = 3391.3 \times 10^{-6}$ m$^4$ and section area $28.84 \times 10^{-3}$ m$^2$. It has a vertical depth of 851 mm and forms part of a floor with concrete slabs firmly bonded to the top 1.5 m wide and 1200 mm thick.

Calculate the stress in the steel at levels (1) and (2) and in the concrete at levels (2) and (3) when a bending moment of 50 kNm is applied to the section. The top layer is in compression. The modulus of elasticity is 205 GPa or steel and 18GPa for concrete.

Change timber into steel. Effective width $B_t = 1.5 \times \frac{E_c}{E_s} = 0.132$ m

For part 1 (I section)

$I_{xx} = 3391.3 \times 10^{-6}$ m$^4$

$A = 0.028.84$ m$^2$.

First moment of area about base $= AH/2 = 0.012$ m$^3$

For part 2 (timber)

$A = BT = 0.026$ m$^2$

First moment about base $= A(H + T/2)$

Total First Moment $= 0.012 + 0.025 = 0.037$ m$^3$

$\bar{y} = 0.037/0.055 = 0.676$ m

SECOND MOMENT OF AREA

For part 2

$I = BT^3/12 = 87.8 \times 10^{-6}$ m$^4$

About the axis $gg$ $I_{gg} = I + A(H + T/2 - \bar{y})^2 = 2.075 \times 10^{-3}$ m$^4$

For part 1

$I_{gg} = I_{xx} + A(\bar{y} - H/2)^2 = 5.206 \times 10^{-3}$ m$^4$

Total $I_{gg} = 7.281 \times 10^{-3}$ m$^4$

Stress in steel at level 1 $\sigma = M \bar{y} / I_{gg} = 50000 \times 0.676/7.281 \times 10^{-3} = 4.64$ MPa

Stress in steel at level 2 $\sigma = M \bar{y} / I_{gg} = 50000 \times 0.175/7.281 \times 10^{-3} = 1.2$ MPa

Stress in concrete at level 2 $\sigma_c = \sigma_s (E_c/E_s) = 1.2 \times 18/205 = 0.105$ MPa

Stress in steel at level 3 $\sigma = M \bar{y} / I_{gg} = 50000 \times 0.375/7.281 \times 10^{-3} = 2.57$ MPa but it is concrete so convert

Stress in concrete at level 3 $\sigma_c = \sigma_s (E_c/E_s) = 2.57 \times 18/205 = 0.226$ MPa

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